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An 'Incomplete Contract' Approach to Bankruptcy  
and the Financial Structure of the Firm

By

Philippe Aghion  
and  
Patrick Bolton

Number 484

March 1988

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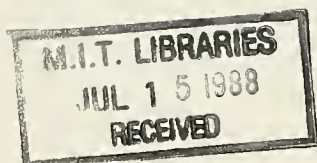
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AN 'INCOMPLETE CONTRACT' APPROACH TO BANKRUPTCY  
AND THE FINANCIAL STRUCTURE OF THE FIRM

By

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## Abstract

In contrast to Modigliani and Miller, who distinguish between debt and equity contracts only in terms of the return-streams of the two types of assets, we emphasize the differences between debt and equity in terms of the rights of control the two assets give investors. Our theory of the firm's financial structure is based on the following considerations: Suppose that the firm must raise external funds to finance an investment. Ideally, the owners of the firm would like to issue non-voting shares, but this is usually unacceptable to outside investors since this amounts to giving full control to the initial owners. Two options are then open: Either issue debt and face the risk of bankruptcy (this involves a transfer of control from the owners to the outside investors) or issue equity and dilute their ownership rights. Both types of assets involve different allocations of control among investors and owners. We argue that the choice of control allocation determines the financial structure of the firm.



## I. Introduction.

Ever since Modigliani and Miller proved their famous irrelevance theorem, the question of the optimal financial structure of the firm has preoccupied economists and, as yet, after nearly thirty years of sustained research, the "capital structure puzzle" has still not been fully resolved. As Myers pointed out in his presidential address to the American Financial Association in 1984:

...we know very little about capital structure. We do not know how firms choose the debt, equity or hybrid securities they issue. We have only recently discovered that capital structure changes convey information to investors. There has been little if any research testing whether the relationship between financial leverage and investors' required return is as the pure MM theory predicts. In general, we have inadequate understanding of corporate financing behavior, and of how that behavior affects security returns...

Myers [1984, p.575].

This paper, we hope, will contribute to the understanding of the firm's financial structure, by pointing out that the "puzzle" partly comes from the way in which the question was originally posed by Modigliani and Miller. They distinguished debt and equity contracts only in terms of the return-streams of the two types of assets and ignored all other differences between these assets. On the other hand, we emphasize the differences between debt and equity in terms of the rights of control the two assets give investors. We attach less importance to the differences in return streams. This leads us to a theory of the financial structure of the firm based on the problem of allocation of control among the investors of the firm.

More specifically, Modigliani and Miller as well as most writers on the firms' financial structure have considered the following basic set-up: suppose that a given project is financed through  $B$  one-dollar bonds and  $E$  one-dollar shares. Assume that this project yields a random return of  $\hat{X}$ .

Then a bond is defined to be an asset that yields a certain return of  $r$  and a share is defined to be an asset that yields a random return of  $\max \left\{ \frac{\hat{X} - rB}{E}; 0 \right\}$ . Within this set-up, Modigliani and Miller have shown that if investors can trade those bonds and shares in a perfect capital market, then any ratio  $B/E$  is an equally efficient means of financing the firm (for a formal proof of the Modigliani-Miller theorem, see Stiglitz [1969]). This result has later been generalized by allowing the firm to go bankrupt (see, for example, Hellwig [1981] and the references therein). [What is usually referred to as "bankruptcy" in these models is simply a limited liability constraint of the firm: if  $\hat{X} < rB$ , then the return on bonds is  $\frac{\hat{X}}{B}$ .]

Since Modigliani and Miller's original contribution, the research agenda has basically concentrated on bringing additional elements into the basic model described above with the objective of obtaining a determinate debt equity ratio. Thus, tax arguments along with costs of bankruptcy arguments have been introduced to yield a determinate financial structure. (See Miller [1977] for a discussion and critique of this approach). More recently, a number of explanations have been developed based on considerations of asymmetric information. According to these theories, the firm's financial structure serves as an incentive scheme for management. (See, for example, Jensen-Meckling [1976], Grossman-Hart [1982], and Ross [1977]). This line of research has, no doubt, brought useful insights. Still, it is widely recognised that all of these explanations suffer from major weaknesses. For example, Hart and Holmström have pointed out that the asymmetric information approach "beg(s) the question why capital structure needs to be used for incentive purposes when direct incentive-schemes would appear cheaper" (Hart-Holmström [1985, p. 24].)

We depart from this line of research in several important ways. First, we do not restrict the firm's choice of assets to only bonds and shares, as defined above. We allow the firm to issue any asset with a general



return-stream  $v(\hat{X})$ , where  $v(\cdot)$  can be any (non-negative) real-valued function. With such a general specification it is no longer possible to distinguish between debt and equity simply by looking at the return-stream of the asset. This brings us to the second distinguishing feature of our approach. In practice, a common-stock contract gives the investor a voting right for every share purchased, whereas debt-contract does not. On the other hand, a debt-contract gives an ownership right to the creditors when the firm goes bankrupt. Thus, equity allocates the ownership of the firm to shareholders so long as they meet the debt-repayments; otherwise, ownership is transferred to creditors. Accordingly, we distinguish between debt and equity in terms of the different control-rights they give to investors. Following Grossman-Hart [1986], we argue that the notion of ownership and control only gains substance, if investment contracts are incomplete and if there are potential conflicts among investors. Thus, although we allow the firm to issue assets with a general return-stream  $r(\hat{X})$ , we assume that the firm cannot write completely contingent contracts. For example, the contract cannot be made contingent on all future actions or all future events.

This approach leads us to a new interpretation of bankruptcy. As we pointed out earlier, economists usually assimilate bankruptcy with limited liability. However here, bankruptcy is a mechanism for transferring ownership from shareholders to creditors. This means, in particular, that bankruptcy is not synonymous with liquidation. Once the creditors gain the ownership of the firm, they can decide to either liquidate the firm or reorganise it. It is well known that the bankruptcy laws in most western developed countries allow for both types of procedures. In the U.S., for example, liquidation falls under chapter 7 of the bankruptcy code and reorganisation under chapter 11. (For an exposition of U.S. bankruptcy law, see White [1984] and Jackson [1985]).

Within this set-up we develop a theory of the firm's financial structure based on the following trade-off. Suppose that a firm needs to raise outside funds to finance a given investment project. Ideally, the owners of the firm would like to obtain these funds with no strings attached, but this is usually unacceptable to the new investors. If these funds are raised by issuing equity this implies that the old owners will have to share control with the new shareholders (even if the new shareholders have only a minority position they will typically gain some control, for example, through "cumulative voting" (see Bhagat and Brickley [1984]) or minority veto power). In other words, the old owners will have to dilute their ownership rights. (An extreme form of dilution is to let a venture capitalist fund the project). Alternatively, the firm could raise funds by issuing debt, thereby allowing the owners to preserve their full ownership rights (so long as they meet the debt repayments). The flip-side of debt, however, is the threat of bankruptcy and the risk of losing control if the firm has financial difficulties. The more debt is raised, the more acute the danger of bankruptcy. In short, the firm will choose its financial structure by weighing the marginal cost of dilution against the marginal cost of debt.

The paper is organised as follows: Section II lays out the model and derives the optimal financial contract under the assumption that this contract is not renegotiated. Section III deals with renegotiation and section IV discusses extensions and various interpretations of the model.

## II. The Model; and Optimal Contracts Without Renegotiation.

We consider the following contracting problem between two agents: an investor who wants to invest part or all of her funds in some profitable venture is dealing with an entrepreneur who needs to borrow funds to pay for his inputs. For simplicity, we shall assume that the entrepreneur has no initial resources. Once a contract has been signed, there are potentially two

periods of production and trade and then the project is terminated. Let  $S > 0$  be the total cost of inputs and let  $y_1$  and  $y_2$  represent total revenue in periods 1 and 2 respectively. Both  $y_1$  and  $y_2$  are random variables, whose realisations are observable and verifiable. The densities of  $y_1$  and  $y_2$  are given by  $f_1(y_1, \theta)$  and  $f_2(y_2; (\theta; a))$ , where the parameters  $\theta$  and  $a$  respectively denote the state of nature and some action plan that must be taken by the firm in period 1. Let the support of distributions  $f_1$  and  $f_2$  be included in the interval  $[y, \bar{y}]$ . We represent the sequence of events in the time-line below:

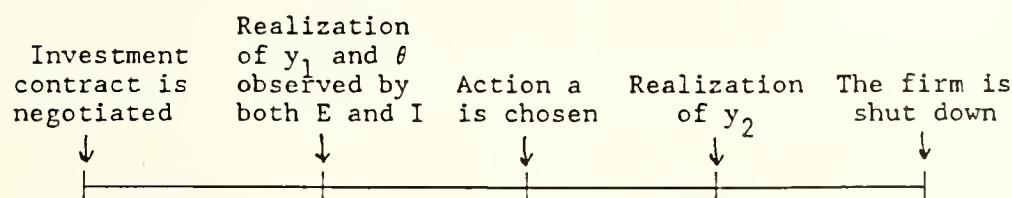


Figure 1

The state of nature,  $\theta$ , should be interpreted very broadly. It could be a change in the law, a change in the weather or even the outcome of an election, etc. For example, the law banning the use of leaded fuel in new cars was an event that drastically affected the distribution of returns of companies producing leaded fuel. We shall assume that there are only two possible states of nature, a "good" state and a "bad" state:  $\theta \in \{\theta_g; \theta_b\}$ . The actions that the firm takes in period 1, we interpret as major management decisions, such as shutting down a plant or reorganising the distribution network, etc. Again, such decisions have an important impact on future returns, so that it is natural to parametrise the densities of  $y_1$  and  $y_2$  by  $\theta$  and  $(\theta; a)$  respectively. Again, for reasons of simplicity, we assume that there are only three possible actions that can be taken in period 1:  $A = \{a_1; a_2; a_3\}$ . To help our intuitive understanding of the model, we attach the following meaning

to each of these actions: choosing action  $a_1$  amounts to running the firm as usual; choosing  $a_2$  is to innovate and, finally, choosing  $a_3$  means to close all operations (i.e., to liquidate the firm).

We assume that both agents are risk neutral, so that the choice of contract by the two parties is not based on any risk-sharing considerations. Next, we assume that the entrepreneur and investor have potentially conflicting objectives. Let  $U(I; a_i)$  be the entrepreneur's Von-Neumann Morgenstern (VNM) utility function over income action pairs and let  $V(I; a_i)$  be the investor's VNM utility function. Then:

$$(1) \quad U(I; a_i) = I - \ell_i \quad i = 1, 2, 3$$

$$(2) \quad V(I; a_i) = I.$$

The conflict between the two parties arises from the fact that the entrepreneur manages the firm and therefore bears private costs (or benefits) in executing the firm's policies. These costs may vary depending on what actions the entrepreneur must carry out. We represent the entrepreneur's private costs by the variable  $\ell_i$ .<sup>1</sup>

We have now set the stage to address the question of how the contract written initially deals with the problem of conflict resolution about what action choice is appropriate in period 1. Of course, the answer to this question will depend on what types of contract are feasible. For example, if the parties can write completely contingent contracts then the initial contract can specify a state-contingent action plan  $a(\theta)$  and there will be no problem of conflict resolution in period 1. In practice, however, it is impossible to write completely contingent contracts so that the parties must confront the problem of the allocation of control in the future. We formalize the impossibility of writing completely contingent contracts by assuming that the initial contract cannot be made contingent on the state of nature and on



actions. The initial contract can only specify profit sharing rules that are contingent on first and second-period revenue-realizations. We denote by  $s_1(\tilde{y}_1)$  and  $s_2(\tilde{y}_1; \tilde{y}_2)$  the share of revenue that the entrepreneur receives in respectively period 1 and period 2. These sharing rules can be completely general except for one important restriction: the limited liability clause. This clause restricts  $s_1(\tilde{y}_1)$  and  $s_2(\tilde{y}_1; \tilde{y}_2)$  to be non-negative. If there were no limited-liability clause, the incompleteness of the contract would be irrelevant. Since he is risk neutral, the first-best could be attained by making the entrepreneur the residual claimant. Thus, without the limited liability restriction the problem would be trivial and uninteresting.

Note that despite the restrictions we have introduced so far, the types of contracts we allow are much more general than most of the contracts considered in the finance literature. In particular, we do not restrict our attention to only standard debt or equity contracts (the former type of contract would impose the restrictions  $s_1(y_1) \cdot y_1 = y_1 - R$ , if  $y_1 \geq R$ , and  $s_1(y_1) = 0$  if  $y_1 < R$  and the latter type of contract would specify  $s_1(y_1) = s_1$  for all  $y_1$ ).<sup>2</sup> Furthermore, since we do not put any a priori restrictions on the return-streams that contracts can specify there is no way of distinguishing debt from equity on that basis. The only way of distinguishing debt from equity in our model is to look at how the initial contract allocates control in period 1.

We consider three possible types of allocations of control: the two polar allocations where either the entrepreneur or the investor has full control and an intermediate case where control to either agent is contingent on the realization of first-period returns.<sup>3</sup> Each type of allocation can be achieved via simple financial contracts which resemble the types of contract observed in practice. Specifically, the investor obtains full control of the firm if the project is financed entirely by issuing equity with voting rights. Since the investor finances the entire project she gets all the shares and

thus obtains full control of the firm. Alternatively, if the entire project is financed via non-voting shares, the entrepreneur retains full control. Finally, a debt contract with a bankruptcy provision or other forms of restrictive covenants leads to an allocation of control where the entrepreneur retains control, provided he can meet his repayment obligations ( $y_1 \geq \bar{y}$ ) or transfers control to the investor when he cannot meet his repayments ( $y_1 < \bar{y}$ ).<sup>4</sup> Of course, one can imagine contracts which make control contingent on  $y_1$  in a different way. We shall discuss when such contracts are appropriate. However, our primary interest is to determine the circumstances under which debt-financing is superior to equity financing with or without voting rights, and when any of these contracts are optimal. Roughly, what determines the mode of financing is the following simple trade off: if the investor controls the firm, she can insure that the firm always pursues a policy that will maximize monetary returns; on the other hand, the investor will abuse her position of control and impose substantial non-pecuniary costs on the entrepreneur by forcing him to take actions that maximise monetary returns. If the entrepreneur controls the firm he can avoid action choices which involve high private costs. The investor will anticipate this and, as a consequence, will raise the cost of investment whenever the entrepreneur has full control. In short, there is a trade-off between the "dilution of power" (letting outside investors participate in the control of the firm) and the cost of investment. We show that debt financing (i.e., making control contingent on first-period revenue realizations) may constitute an efficient compromise since it gives control to each agent in the circumstances when it matters most to have control. To demonstrate this, we proceed as follows: first we derive the efficient contracts when the entrepreneur or the investor has full control. Then we show that under certain conditions a debt contract (which makes control contingent on first-period revenue realizations) may dominate the efficient contracts when either the investor or the entrepreneur



has full control.

a) The Efficient Contract When the Entrepreneur Controls the Firm.

We assume that the entrepreneur sets the contract. He must then solve the following optimisation problem:

$$\int_{\underline{y}}^{\bar{y}} s_1(y_1) y_1 f_1(y_1) dy_1 + q \int_{\underline{y}}^{\bar{y}} s_2(y_1, y_2) y_2 f_2(y_2; (a_i, \theta_g)) dy_2 - \ell_i +$$

max

$$(a_i; a_j) \in A \times A$$

$$s_1(\cdot); s_2(\cdot; \cdot)$$

$$(1-q) \int_{\underline{y}}^{\bar{y}} s_2(y_1; y_2) y_2 f_2(y_2; (a_j; \theta_b)) dy_2 - \ell_j$$

subject to

$$(IR) \int_{\underline{y}}^{\bar{y}} (1-s_1(y_1)) y_1 f_1(y_1) dy_1 + q \int_{\underline{y}}^{\bar{y}} (1-s_2(y_1; y_2)) y_2 f_2(y_2; (a_i; \theta_g)) dy_2 +$$

$$(1-q) \int_{\underline{y}}^{\bar{y}} (1-s_2(y_1; y_2)) y_2 f_2(y_2; (a_j; \theta_b)) dy_2 \geq S$$

$$\begin{aligned}
(IC)_g \quad & \int_y^{\bar{y}} s_2(y_1; y_2) y_2 f_2(y_2; (a_i; \theta_g)) dy_2 - \ell_i \\
& \geq \int_y^{\bar{y}} s_2(y_1; y_2) y_2 f_2(y_2; (a_k; \theta_g)) dy_2 - \ell_k \quad \text{for } k \neq i
\end{aligned}$$

$$\begin{aligned}
(IC)_b \quad & \int_y^{\bar{y}} s_2(y_1; y_2) y_2 f_2(y_2; (a_j; \theta_b)) dy_2 - \ell_j \\
& \geq \int_y^{\bar{y}} s_2(y_1; y_2) y_2 f_2(y_2; (a_k; \theta_b)) dy_2 - \ell_k \quad k \neq j
\end{aligned}$$

$$\text{and (LL)} \quad s_1(\cdot) \geq 0; \quad s_2(\cdot; \cdot) \geq 0.$$

$$\text{where} \quad q \equiv P_r(\theta = \theta_g) \text{ and}$$

$$f_1(y_1) \equiv q f_1(y_1; \theta_g) + (1 - q) f_1(y_1; \theta_b).$$

The constraint (IR) tells us that a contract is acceptable to the investor only if she gets at least an expected return of zero. The constraints  $(IC)_g$  and  $(IC)_b$  guarantee that the entrepreneur will indeed stick to the action plan  $(a_i; a_j)$  in period 1. This program is quite complicated, and we shall proceed to solve it by successive simplifications. First, note that since both agents are risk neutral we can restrict ourselves without loss of generality to sharing rules of the form,  $s_1(y_1) = s_1$  for all  $y_1$ . Second, note that if  $\int_y^{\bar{y}} y_1 f_1(y_1) dy_1 \geq S$ , the first-best outcome can be attained since there is a

feasible contract which makes the entrepreneur the residual claimant in period 2. We shall be interested in the solution when  $\int_y^{\bar{y}} y_1 f_1(y_1) dy_1 < S$ . In this case, we can restrict ourselves without loss of generality to contracts which set  $s_1 = 0$ .

We shall make another useful simplification by only considering second-period sharing rules  $s_2(y_1; y_2) = s_2$ . This restriction, however, involves a loss in generality for the following reasons. For one thing, the particular realization of  $y_2$  may be a good indication of what action was chosen in period 1. Thus, the contract can be made contingent on actions, indirectly, by making  $s_2$  contingent on  $y_2$ . A special case where the restriction  $s_2(y_1; y_2) = s_2(y_1)$  is warranted is when the support of second period revenue is  $(0; 1)$ . We shall pursue our analysis by restricting our attention to this case. In the Appendix, we relax the assumption that contracts cannot be made contingent on actions and show that all our results obtained below can be reproduced even if contracts are contingent on actions. Consequently, the restriction  $s_2(y_2; y_1) = s_2(y_1)$  is not essential for our results.

It is also restrictive to make the second-period sharing rule independent of  $y_1$  for if  $y_1$  is sufficiently correlated with the state of nature  $\theta$ , the constraints  $(IC)_g$  and  $(IC)_b$  are more easily satisfied simultaneously if  $s_2$  takes on different values for different realizations of  $y_1$ . In the extreme case where  $y_1$  is perfectly correlated with  $\theta$ , making  $s_2$  contingent on  $y_1$  is equivalent to writing a contract contingent on  $\theta$ . Again, we analyse contracts where  $s_2$  is contingent on  $y_1$  in the Appendix and the qualitative results obtained here remain unchanged when one allows for the more general class of contracts. With our simplifications we can rewrite the constraints in the above program as follows:

$$(IR) \quad (1-s_2)[q\pi_{gi}+(1-q)\pi_{bj}] \geq k,$$

$$(IC)_g \quad s_2\pi_{gi}^{-l_i} \geq s_2\pi_{gk}^{-l_k} \quad k \neq i,$$

$$(IC)_b \quad s_2\pi_{bj}^{-l_j} \geq s_2\pi_{bk}^{-l_k} \quad k \neq j,$$

$$(\text{where } \pi_{gi} = \int_{\bar{y}} y_2 f_2(y_2 | (a_i; \theta_g)) dy_2 \text{ and } \pi_{bj} = \int_{\bar{y}} y_2 f_2(y_2 | (a_i; \theta_b)) dy_2).$$

One can now easily solve the program by first choosing an arbitrary action plan  $(a_i; a_j)$  and checking if it is feasible. That is, if it satisfies all five constraints for some  $s_2 \in [0,1]$  (the (IR) constraint is violated for all  $s_2 \geq 1$ ). Second, for all the feasible action plans one can determine, the maximum expected returns generated by any such action plan. The optimal action plan(s) are those which yield the highest expected returns to the entrepreneur.

For the sake of illustration and to help our intuition we shall make the following additional assumptions about second period revenues and the private costs of the entrepreneur:

$$(i) \quad \pi_{g2} > \pi_{g1} \text{ and } \pi_{g2} > \pi_{g3}$$

$$(ii) \quad \pi_{g1}^{-l_1} > \pi_{g2}^{-l_2} \text{ and } \pi_{g1}^{-l_1} > \pi_{g3}^{-l_3}$$

$$(iii) \quad \pi_{b3} > \pi_{b1} \text{ and } \pi_{b3} > \pi_{b2}$$

$$(iv) \quad \pi_{b3}^{-l_3} > \pi_{b1}^{-l_1} \text{ and } \pi_{b3}^{-l_3} > \pi_{b2}^{-l_2}.$$

Conditions (i) and (iii) say that the investor's most preferred action plan is  $a_2$  in state  $\theta_g$  and  $a_3$  in state  $\theta_b$ . In other words the investor prefers to liquidate the firm in the bad state  $\theta_b$  and to innovate in the good state,  $\theta_g$ .

Note that the investor's preferences are the same for any second-period sharing rule  $s_2 \in (0,1)$ . Conditions (ii) and (iv), on the other hand, tell us that the efficient action-plan net of the entrepreneur's private costs is to choose  $a_1$  in state  $\theta_g$  and  $a_3$  in state  $\theta_b$ .

When the entrepreneur has full control of the firm, our assumptions imply that the only feasible action in state  $\theta_g$  is  $a_1$ , while any action may be feasible in state  $\theta_b$ .

If  $a_3$  is feasible in state  $\theta_b$ , then the first-best can be attained and this is clearly the efficient solution. Suppose without loss of generality, that only one  $(IC)_b$  constraint is binding at the optimum,<sup>5</sup> and let this constraint be given by:

$$s_2\pi_{b3} - \ell_3 = s_2\pi_{b1} - \ell_1.$$

Then, the first best action,  $a_3$ , is feasible if the following inequality is satisfied:

$$(3) \quad \frac{\ell_3 - \ell_1}{\pi_{b3} - \pi_{b1}} \leq 1 - \frac{k}{q\pi_{g1} + (1-q)\pi_{b3}}.$$

This inequality will be violated if, for example,  $\ell_3$  or  $k$  is large. If the private costs of liquidating  $\ell_3$  are large, the entrepreneur must receive a large fraction of the monetary returns of the firm, for him to choose  $a_3$ . But this implies that the investor might get such a small fraction of revenue that she will refuse to invest. It is not altogether surprising that the first-best plan is not implementable in general when the entrepreneur has full control of the firm. The basic point is that since the entrepreneur has no funds and since first-period expected revenues are relatively small compared with the initial investment, the entrepreneur cannot be made the residual claimant of the firm. Consequently, he will not choose the first-best action plan in general.

When inequality (3) is not satisfied only  $a_1$  and  $a_2$  can be feasible in state  $\theta_b$ . We shall assume that the optimal solution then is given by the action plan  $(a_1; a_1)$ . In other words, we assume that when the entrepreneur has full control he will choose to run the firm as usual in period 2, no matter what the circumstances are.<sup>6</sup> His expected payoff is then given by:

$$\pi_{NVS} = q\pi_{g1} + (1 - q)\pi_{b1} - \ell_1 - k,$$

(where  $\pi_{NVS}$  stands for expected profits when the firm is financed with non-voting shares (NVS).)

b) The Efficient Contract When the Investor Controls the Firm.

We proceed here as in section 11a) by restricting the contract to simple sharing rules  $s_1$  and  $s_2$ . The program now facing the entrepreneur is:

$$\begin{aligned} \max \quad & s_1 \int_y^{\bar{y}} y_1 f_1(y_1) dy_1 + s_2 [q\pi_{gi} + (1 - q)\pi_{bj}] \\ s_1 \geq 0; \quad & s_2 \geq 0 \\ (a_i, a_j) \in & A \times A \end{aligned}$$

$$\text{subject to (IR):} \quad (1 - s_1) \int_y^{\bar{y}} y_1 f_1(y_1) dy_1 + (1 - s_2) [q\pi_{gi} + (1 - q)\pi_{bj}] \geq S$$

$$(IC)_g \quad (1 - s_2)\pi_{gi} \geq (1 - s_2)\pi_{gk} \quad k \neq i$$

$$(IC)_b \quad (1 - s_2)\pi_{bj} \geq (1 - s_2)\pi_{bk} \quad k \neq j$$

Given our assumptions it is straightforward to see that the only implementable action plan is the pair  $(a_2; a_3)$ , and this yields an expected payoff for the entrepreneur of:

$$\pi_E = q(\pi_{g2} - \ell_2) + (1 - q)(\pi_{b3} - \ell_3) - k.$$



When the investor controls the firm, she chooses the actions that yield the highest monetary returns. These actions are,  $a_2$  in state  $\theta_g$  and  $a_3$  in state  $\theta_b$ .

Our discussion so far reveals that when inequality (3) is violated and when the firm is entirely financed by issuing non voting shares, then the firm chooses the efficient course of action in state  $\theta_g$  but chooses an inefficient action in state  $\theta_b$ : instead of liquidating the entrepreneur prefers to keep operating the firm. On the other hand, when the firm is entirely financed by issuing voting shares then it will choose the efficient action in state  $\theta_b$  but not in state  $\theta_g$ : in the latter state the investor chooses the Pareto-inferior action which maximizes her monetary returns. We can easily determine which mode of financing is better:

*Proposition 1:* Voting shares dominate non-voting shares when the following two inequalities are satisfied:

$$(a) \quad \frac{\ell_3 - \ell_1}{\pi_{b3} - \pi_{b1}} > 1 - \frac{k}{q\pi_{g1} + (1 - q)\pi_{b3}}$$

$$(b) \quad q[(\pi_{g2} - \ell_2) - (\pi_{g1} - \ell_1)] + (1 - q)[(\pi_{b3} - \ell_3) - (\pi_{b1} - \ell_1)] > 0$$

The first inequality guarantees that the first-best cannot be attained with a non-voting equity contract. The second inequality simply says that  $\pi_E > \pi_{NVS}$  (when the first condition is satisfied). Recall that  $\pi_{g2} - \ell_2 < \pi_{g1} - \ell_1$  and  $\pi_{b3} - \ell_3 > \pi_{b1} - \ell_1$ ; thus voting-shares dominate non-voting shares when the inefficiency resulting in state  $\theta_g$  is (substantially) smaller than the inefficiency resulting in state  $\theta_b$ . Alternatively, voting shares dominate non-voting shares, when control matters more for the investor than for the entrepreneur.

c) Contracts Where Control is Contingent on  $y_1$ .

We have established that (under certain conditions) allocating control entirely to one party inevitably results in some inefficient action-choice in one of the two states  $(\theta_g; \theta_b)$ . In addition, our model has the feature that the inefficient action-choice takes place in a different state when the entrepreneur controls the firm than when the investor controls the firm. This suggests that if first-period revenue is sufficiently correlated with the state of nature, one could improve upon the types of contract considered so far by making control contingent on first-period revenue realisations. Indeed, in the extreme case where  $y_1$  is perfectly correlated with  $\theta$ , control can be made contingent on the state of nature and the allocation which gives control to the entrepreneur in state  $\theta_g$  and to the investor in state  $\theta_b$  achieves the first-best. When  $y_1$  is imperfectly correlated with  $\theta$ , it is less obvious that contracts with a contingent control-allocation rule dominate the types of contract considered previously. We begin by considering the following contingent control allocation rule: if  $y_1 < \hat{y}$  then the investor controls the firm; otherwise, the entrepreneur maintains full control. Such rules are often observed in practice. For example, a bankruptcy mechanism which forces management and shareholders to give up the firm's assets to creditors when the firm cannot meet its debt repayments is of this form. Admittedly, the above allocation rule is somewhat extreme. In practice, when a firm's performance deteriorates, its shareholders and management hand over control to creditors gradually and bankruptcy is only the final outcome of a long process.

The next proposition establishes a sufficient condition for contracts with the above contingent allocation rule to dominate non-voting share contracts and voting share contracts:

*Proposition 2:* Let  $F(y_1|\theta_g)$  and  $F_1(y_1|\theta_b)$  be the conditional distribution functions of first-period revenue, then if

$$(a) \quad \frac{\ell_3 - \ell_1}{\pi_{b3} - \pi_{b1}} > 1 - \frac{k}{q\pi_{g1} + (1-q)\pi_{b3}}$$

$$(b) \quad \frac{F(y_1|\theta_b)}{F(y_1|\theta_g)} \text{ is sufficiently large for the lowest realisations of } y_1$$

(i.e., for  $y_1$  close to  $\hat{y}$ ) then a contract which allocates control to the investor when  $y_1 < \hat{y}$  (some  $\hat{y}$ ) and otherwise to the entrepreneur dominates any contract which does not make control contingent on  $y_1$ .

*Proof:* When condition (a) is verified, we know that the entrepreneur would choose the pair of actions  $(a_1; a_1)$ . We also know that when the investor controls the firm she chooses the pair  $(a_2; a_3)$ . Thus, the expected payoff to the entrepreneur of a contract with the above contingent allocation rule is given by:

$$\begin{aligned} \pi_D = & q((1 - F(\hat{y}|\theta_g))(\pi_{g1} - \ell_1) + F(\hat{y}|\theta_g)(\pi_{g2} - \ell_2)) + \\ & (1 - q)((1 - F(\hat{y}|\theta_b))(\pi_{b1} - \ell_1) + F(\hat{y}|\theta_b)(\pi_{b3} - \ell_3)) - k \end{aligned}$$

We can assume without loss of generality that  $\pi_{NVS} \geq \pi_E$ . Thus, we want to show that  $\pi_D > \pi_{NVS}$ , when  $F(\hat{y}|\theta_b)/F(\hat{y}|\theta_g)$  is sufficiently large. Now, the inequality  $\pi_D > \pi_{NVS}$  can be written as follows:

$$\frac{F(\hat{y}|\theta_b)}{F(\hat{y}|\theta_g)} \geq \frac{q[(\pi_{g1} - \ell_1) - (\pi_{g2} - \ell_2)]}{(1 - q)[(\pi_{b3} - \ell_3) - (\pi_{b1} - \ell_1)]}$$

Q.E.D.

Proposition 2 defines one sufficient condition, but one can think of many others. The basic requirement is that  $y_1$  be a sufficiently good signal for

the state of nature over some range. We have not picked the interval of the lowest revenue-realizations  $[y, \hat{y}]$  ( $\hat{y} > y$ ) at random. In practice, it is often the case that when the firm's performance is bad this indicates that the net-present value of the firm is low (in other words, that the firm is in a "bad state"). Although one can conceive of distribution-functions  $F(y_1|\theta)$  such that high realizations of  $y_1$  are a good signal of the state  $\theta_b$ , it is hard to find situations in the real world which correspond to this case. It is this observation which leads us to choose the condition in Proposition 2. Also, a contract which allocates control to the creditor for low revenue realisations can usefully be interpreted as a debt-contract with a bankruptcy provision. We are primarily interested in this interpretation, and the fact is that most debt contracts only transfer control to creditors when the firm's performance is bad. We rarely see successful firms being liquidated by creditors.

The next proposition gives a condition which guarantees the optimality of the contingent allocation-of-control rule encountered in most debt contracts: We say that the density  $f(y_1|\theta)$  satisfies the monotone likelihood ratio property (MLRP) if the ratio  $f(y_1|\theta_g)/f(y_1|\theta_b)$  is increasing in  $y_1$ .

*Proposition 3:* If  $f(y_1|\theta)$  satisfies MLRP and if conditions (a) and (b) in Proposition 2 are met, then a contract which allocates control to the entrepreneur when  $y_1 \geq \hat{y}$  (some  $\hat{y} \in (y, \bar{y})$ ) and to the investor when  $y_1 < \hat{y}$  is optimal.

*Proof:* Consider a contract with the following general contingent allocation rule: Let  $I, J$  be two subsets of the interval  $[y, \bar{y}]$  ( $I \cap J = \emptyset$  and  $I \cup J = [y, \bar{y}]$ ), such that if  $y_1 \in I$ , the investor controls the firm and if  $y_1 \in J$ , i.e.,  $y_1 \in J$ , the entrepreneur controls the firm. We will show that if  $I$  is not equal to an interval of the form  $[y, \bar{y}]$  where  $y > y$ , then the contract is

suboptimal. Thus, suppose  $I \neq [y, y]$ . We can find  $\hat{y}$  such that

$$(4) \quad F(\hat{y} | \theta_g) = \text{Prob}(y_1 \in I | \theta_g).$$

Since  $I \neq [y, \hat{y}]$ , we have  $\hat{y} < \sup_{y_1 \in I} y_1$ .

Now, denote by  $\pi_I$  the expected payoff to the entrepreneur from the contract giving him control whenever  $y_1 \notin I$  and denote by  $\pi_{\hat{y}}$  the expected payoff to the entrepreneur when he gets control for all  $y_1 \notin [y, \hat{y}]$ . Then we obtain:

$$\pi_I - \pi_{\hat{y}} = (1 - q) \{ [\text{Prob}(y_1 \in I | \theta_b) - F(\hat{y} | \theta_b)] [(\pi_{b3} - \ell_3) - (\pi_{b1} - \ell_1)] \}.$$

If  $\text{Prob}(y_1 \in I | \theta_b) < F(\hat{y} | \theta_b)$  we have established our claim. But the latter inequality simply follows from (4) and MLRP. Q.E.D.

If  $y_1$  is a poor signal of the state  $\theta$ , a contract with a contingent allocation of control rule may be dominated by a contract which gives full control to either of the parties. Thus, the allocation of control in the firm will depend first on the configuration of preferences and second on the degree of correlation between  $y_1$  and  $\theta$ . We have identified situations where it is optimal to give full control to the entrepreneur, others where it is optimal for the investor to control the firm and finally situations where the optimal arrangement is to have control allocated contingently on  $y_1$ . Any of these allocations can be implemented by writing standard financial contracts: full control to the entrepreneur is attained by issuing non-voting shares; full control to the investor is achieved by issuing voting shares and finally (when  $f(y_1 | \theta)$  satisfies MLRP) control can be allocated contingently on  $y_1$ , by writing a debt-contract with a bankruptcy provision. (Of course, other contract forms can also do the job, and we shall discuss some of them in section IV). Thus, our discussion so far indicates that the determination of the financial structure of the firm can be viewed as a problem of allocation



of control. The main purpose of our paper is to show how control rights attached to various types of financial assets can explain the determination of the financial structure of the firm.

If we interpret the allocation rule described in Proposition 3 as a bankruptcy mechanism we obtain the interesting insight that bankruptcy is not equivalent to liquidation. Since  $y_1$  may not be perfectly correlated with  $\theta$ , bankruptcy can occur with positive probability in the good state. In other words, the creditor can get control of the firm in the good state, in which case she will choose not to liquidate (i.e., not select  $a_3$ ) but rather to reorganise (by selecting  $a_2$ ). In the U.S. it is indeed possible that bankruptcy may either lead to liquidation (chapter 7) or reorganisation (chapter 11). Economists usually assimilate bankruptcy to liquidation and ignore the possibility of reorganisation (a notable exception is M.White [1984]): Bankruptcy is not viewed as a mechanism for transferring control to creditors. Rather, it is viewed as the interruption of a production activity when the net present value of liquidation is higher than the net present value of continuation. Given this interpretation it is hard to understand why bankruptcy should ever lead to reorganisation: If it were established in the first place that the net present value of the firm is higher under liquidation than continuation why reorganise? Or, if continuation is more profitable under a new investment plan than liquidation, why is bankruptcy and reorganisation necessary to implement this new plan?

This brings us to the question of renegotiation. One interpretation of bankruptcy and reorganisation is that it constitutes a convenient institution to bring all investors and claimants to the negotiating table. That is to say, bankruptcy is not so much a mechanism of transfer of control to creditors as a forum where investors renegotiate their claims on the revenues of the firm. We shall take up this interpretation in more detail in the next section.



### III. Renegotiation.

The previous section assumed that the parties could not renegotiate the contract after the realization of the state of nature. It was shown that without ex-post renegotiation an optimal contract may specify an allocation of control which is contingent on first-period revenue realizations. Bankruptcy can be interpreted as such an allocation of control rights and it was argued that one of the roles of bankruptcy is to serve as a mechanism for transferring control away from shareholders to creditors. This contrasts with another common interpretation which views bankruptcy as a negotiation forum where all investors either renegotiate their claims or settle the distribution of the proceeds from liquidation. Undoubtedly, one function of the bankruptcy institution is to bring all the parties involved in the firm to the negotiating table. It is thus clearly unsatisfactory to rule out ex-post renegotiation. In this section we look at optimal contracts when the parties recognize that the initial contract can be renegotiated ex-post. The general questions we address are: (1) whether allocation of control ex-ante remains an important issue when contracts can be renegotiated ex-post? We show that, indeed, under certain conditions the question of allocation of control is vacuous, when contracts can be renegotiated ex-post. However, our main point in this section is to show that this is not generally the case. Thus, the second question we address is: (2) If the allocation of control in the initial contract matters, is it still the case that a mechanism like bankruptcy is optimal?

It is not our purpose in this section to provide a full-fledged analysis of contracting with renegotiation. Such an undertaking is beyond the scope of this paper. Our objective here is to convince the reader that our interpretation of bankruptcy as a mechanism for transferring control does not depend on the assumption that the initial contract cannot be renegotiated.

To make our point, we restrict attention to the simplest renegotiation game. We assume that the entrepreneur can make a take-it-or-leave-it offer of a new contract to the investor after the realization of the state of nature. If the latter accepts the offer the initial contract is rescinded, otherwise the parties stick to the terms in the initial contract. The timing of moves we consider is illustrated in the time-line below:

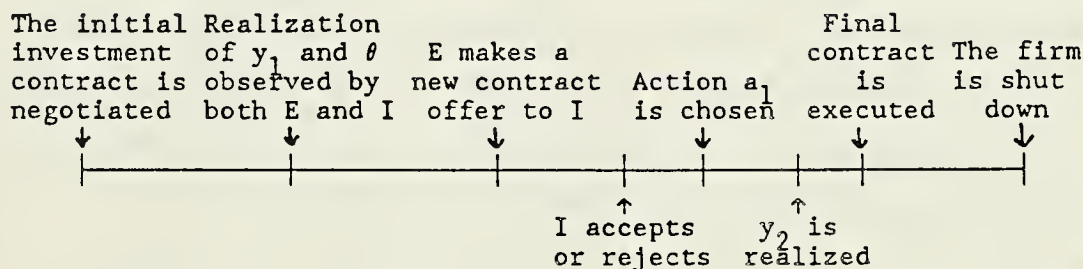


Figure 2

As in Section II, the initial contract specifies a sharing-rule of second-period revenues,  $s$ , and an allocation of control. Given our assumption about the entrepreneur's bargaining power ex-post, it will never be feasible to invest first and wait until the realization of the state of nature before negotiating a contract, for then the investor will always make a loss.

To begin with, we shall illustrate how renegotiation ex-post can lead to the implementation of the first-best outcome and how, as a result, the ex-ante allocation of control may become irrelevant. First, consider the case where the initial contract gives full control to the entrepreneur and condition (a) of proposition 2 is satisfied. In this case the optimal contract without renegotiation implements the action pair  $(a_1; a_1)$  and the investor's share of second-period revenue is given by:

$$1 - s = \frac{k}{q\pi_{g1} + (1 - q)\pi_{b1}}.$$

Suppose now that state  $\theta_b$  is realized. In this state the first-best action is  $a_3$ , but the initial contract induces the entrepreneur to choose  $a_1$ . He would be willing, though, to choose  $a_3$  provided he receives some transfer  $t_0$  from the investor, where:

$$(5) \quad s \cdot \pi_{b3} - \ell_3 + t_0 \geq s \cdot \pi_{b1} - \ell_1$$

On the other hand the investor will find it profitable to make such a transfer if and only if:

$$(6) \quad (1 - s) \cdot \pi_{b3} - t_0 \geq (1 - s) \cdot \pi_{b1}$$

Now the existence of a transfer  $t_0 \geq 0$  that satisfies both conditions (5) and (6) simply follows from the inequality:

$$\pi_{b3} - \ell_3 > \pi_{b1} - \ell_1$$

which is verified by assumption. So, renegotiation will actually take place at date 1 in state  $\theta_b$  and induce the entrepreneur to choose the first-best action  $a_3$  in that state. In addition, renegotiation *ex-post* does not affect *ex-ante* investment decisions. Consequently, the first-best can always be attained by giving full control to the entrepreneur.

Now consider the case where the initial contract gives full control to the investor. Then we know from section II, that the optimal contract without renegotiation implements the action pair  $(a_2, a_3)$  and the investor's share of second period revenue is given by:

$$1 - s = \frac{k}{q\pi_{g2} + (1 - q)\pi_{b3}}$$

When state  $\theta_g$  is realized, the investor would be willing to switch from action  $a_2$  to  $a_1$  if the entrepreneur offers a new sharing-rule of second period

profits,  $\hat{s}$ , such that:

$$(1 - s)\pi_{g2} \leq (1 - \hat{s})\pi_{g1}$$

The entrepreneur would offer to renegotiate the contract if

$$\hat{s}\pi_{g1} - \ell_1 \geq s\pi_{g2} - \ell_2 \text{ and } \hat{s} \geq 0.^7$$

Given that  $\pi_{g1} - \ell_1 > \pi_{g2} - \ell_2$ , all these conditions are satisfied whenever:

$$\frac{k}{q\pi_{g2} + (1-q)\pi_{b3}} \cdot \frac{\pi_{g2}}{\pi_{g1}} \leq 1.$$

Thus, renegotiation here could also lead to the implementation of the first-best outcome. This illustrates not only how renegotiation *ex-post* can lead to the implementation of the first-best outcome but also how the issue of allocation of control may become irrelevant when contracts can be renegotiated freely, *ex-post*. It is not true, however, that renegotiation always leads to the first-best. Also, the allocation of control can be a relevant issue, since it can affect the outcome of the renegotiation process. We shall now provide an example to illustrate how contracting with renegotiation may not implement the first-best outcome. In this example it is assumed that

1) condition (a) in proposition 2 holds.

$$2) \quad q\pi_{g1} + (1-q)\pi_{b1} < k$$

$$3) \quad \frac{k}{q\pi_{g2} + (1-q)\pi_{b3}} \cdot \frac{\pi_{g2}}{\pi_{g1}} > 1$$

4)  $(1 - \bar{s})[q\pi_{g2} + (1 - \bar{s})\pi_{g3}] > k$ , where  $\bar{s}$  is given by:

$$q[\bar{s}\pi_{g2} - \ell_2] + (1 - q)[\bar{s}\pi_{g3} - \ell_3] = 0.$$

The first condition ensures that if the initial contract gives full control to

the entrepreneur and if this contract is not renegotiated then the entrepreneur picks the action plan  $(a_1; a_1)$ .

Now suppose that the initial contract gives full control to the entrepreneur, and that the initial contract specifies some sharing rule  $s \geq 0$ . Given that the entrepreneur has all the bargaining power in the negotiation phase, he can hold the investor's expected return down to  $(1 - s)[q\pi_{g1} + (1 - g)\pi_{b1}]$  (this is the payoff the investor can expect, when she rejects the entrepreneur's new contract offer in state  $\theta_b$ ). Condition 2) then implies that giving full control to the entrepreneur in the initial contract cannot be acceptable to the investor. The latter must be given some control-rights so as to guarantee a minimum expected return which makes the investment profitable. Consequently the only feasible initial contracts are those which give partial or full control to the investor.

Consider first the initial contract which gives full control to the investor. We know that if such a contract is not renegotiated, the investor chooses the action plan  $(a_2; a_3)$ . Condition 4) tells us that such a contract is acceptable for both the investor and the entrepreneur. Furthermore, condition 3) implies that renegotiation is not feasible in state  $\theta_g$ : Because of the limited liability constraint, the entrepreneur cannot find a new contract which is acceptable to the investor and induces her to choose action  $a_1$  instead of  $a_2$ . Thus, the first-best outcome cannot be implemented when conditions 1) to 4) are satisfied.

Now if first-period revenues are a sufficiently good signal of the state of nature, then an initial contract which specifies a state-contingent allocation of control will dominate a contract which gives full control to the investor. This is obvious when first-period revenue is perfectly correlated with the state of nature for then the initial contract implements the first-best while the contract which gives full control to the investor does not. For this example one can now easily derive conditions on the



distribution of  $y_1$  similar to those specified in proposition 3, which guarantee that a debt-contract is optimal even when contracts can be renegotiated after the realization of the state of nature.

To summarize, our limited liability assumption limits the extent of renegotiation *ex-post*. As a result, renegotiation does not always lead to an *ex-post* efficient solution. The parties must then confront the issue of the optimal allocation of control in the initial contract. Whenever  $y_1$  is a sufficiently good signal of  $\theta$ , it may be optimal to make control contingent on  $y_1$ . In other words, it may be optimal to write a debt contract with a bankruptcy mechanism even when renegotiation *ex-post* is allowed.

#### IV. Interpretations and Extensions.

We have developed a highly stylized model to address the issue of allocation of control in closely held firms. As was explained in section II, this issue arises when contracts between the investor and the entrepreneur are incomplete and when the preferences of the two agents diverge. Given the incompleteness of the contract and the potential conflict between the two parties, the initial contract must specify who will have the authority to make future decisions. We have shown that the allocation of control ultimately specified in the initial contract will depend on the particular form of the entrepreneur and investor's preferences and on the degree of correlation between the firm's revenues and the state of nature. At this level of abstraction, this is not a very insightful conclusion. However, the model gains substance when one draws analogies between any particular initial contract written by the investor and entrepreneur and standard financial contracts observed in practice. Thus, we have shown that most allocations of control can be implemented by choosing the appropriate financial structure for the firm: if it is optimal to give full control to the investor, the firm should finance its investment by issuing voting-shares; if it is best for the



entrepreneur to retain full control, the firm should issue non-voting shares and finally if the efficient arrangement is to allocate control to the entrepreneur in the "good states of nature" and to the investor in the "bad states of nature," then the firm ought to (partially) finance its investment by issuing debt so that control can be shifted from the entrepreneur to the creditors through the bankruptcy mechanism. (Of course, control could also be shifted from the entrepreneur to the investor by means other than the bankruptcy mechanism, and we shall take up the discussion of such alternative schemes later on).

Drawing these analogies has allowed us to develop an alternative interpretation of both the determination of the financial structure of the firm and the role of bankruptcy. In a nutshell, the determination of the firm's financial structure can be viewed as a problem of allocation of control and the role of bankruptcy can be interpreted as serving the purpose of transferring control from shareholders to creditors. Under this interpretation, there is no necessary connection between bankruptcy and insolvency. Bankruptcy can result in either reorganisation or liquidation. The important point is that after bankruptcy the shareholders have lost control and the creditors take over. Our interpretation constitutes a significant departure from existing theories of the financial structure of the firm and from existing studies of bankruptcy. We discussed the main features of some of the most influential existing theories of the financial structure in the introduction. The subsequent discussion, therefore, mainly emphasizes the links between our work and some of the economic literature on bankruptcy.

A common feature in most of the existing models of bankruptcy (in particular, Bulow-Shoven [1978], Green-Shoven [1984], and White [1980]) is that the bankruptcy decision is analysed given the assets and liabilities of the firm. No attempt is made at modeling the initial financing decisions of the firm. The primary concern of these papers is to study whether and when

the creditors' decision to liquidate the firm is efficient. Bankruptcy is defined to be a situation of insolvency, and the possibility that bankruptcy may lead to reorganisation is ruled out. Thus, the fact that bankruptcy transfers control to creditors is not emphasized. A recent paper that looks at the bankruptcy decision from an ex-ante point of view within a multi-period model is Bray [1986]. Some of the main points of her paper are that in a multi-period setting debt contracts can be more flexible than is usually assumed and also that bankruptcy is not just a decision between liquidation and continuation, but a decision between various ways of continuing the firm and liquidation. However, Bray does not take an incomplete-contracts approach; instead she restricts the set of contracts to standard multi-period debt contracts.

Our approach is more general to the extent that we do not put any *a priori* restrictions on financial contracts other than the restriction that the contract cannot be made contingent on the state of nature. (Without this latter restriction the question of allocation of control and, for that matter, of the optimal financial structure of the firm is not well defined). This allows us to determine the financial structure of the firm, given the configuration of preferences of the entrepreneur and investor. We have shown that debt contracts are not always optimal. In some circumstances equity financing may dominate debt-financing. Then, of course, the problem of bankruptcy does not even arise.

Although our work is more general, it still suffers from major shortcomings. At best, it applies only to closely held firms, since our model has only one investor who does not trade her assets with other investors. We hope to extend the model to several investors in future work. It is widely held that the main purpose of bankruptcy law is to resolve conflicts among creditors (for a recent exposition of this view, see Baird [1986] or Baird and Jackson [1985]). However, this claim has not yet received a satisfactory

formal treatment. It seems, therefore, important to develop a model of a firm with several investors and investigate whether:

- (1) initial investment contracts are efficient at resolving the potential conflict among creditors; and, if not,
- (2) whether the bankruptcy institution, as we know it, is an efficient supplement to the investment contracts.

These are by no means obvious questions. For example, Baird [1986] is questioning the necessity of rules governing corporate reorganisation altogether. His argumentation is compelling, and the questions raised in his paper should be addressed in a model similar to ours with several investors.

Another weakness of our analysis is that although we can make a case for a mechanism like the bankruptcy process, we cannot explain why a special bankruptcy law is required. In other words, we have not addressed important enforcement issues, which no doubt must be part of any explanation of bankruptcy. Again, the question of why we need a bankruptcy law is difficult. The legal writers who view the firm as originating from a bargaining process among investors are the first to admit that it is still unclear what the exact function of the bankruptcy law is. Thus, for example, Baird writes:

The view that the goal of bankruptcy law governing corporate debtors is to solve a common pool problem explains the general shape it should have, but it does not explain many of its details. As with any off-the-rack term, it is difficult to tell whether bankruptcy law ... would in fact be part of an investor's bargain.

Baird [1985, p. 135].

It is remarkable that the bankruptcy law in its general form is very similar in most developed capitalist economies. Thus, for example in the U.S., Germany, France and Japan the law specifies two sets of rules: one for liquidation and one for reorganisation. In addition, the law provides for some form of receivership in each of these countries. The major differences

concern the treatment of various classes of creditors and other claimants like workers and trade creditors (for an exposition of the main features of bankruptcy law in Japan, see Kitagawa [1987]; in France, Simeon et al. [1987]; in Germany, Rüster [1987] and for a discussion of U.S. law, see Jackson [1986] and White [1984]). Naturally, there may also be considerable differences in the way the law is implemented.

Finally, another general shortcoming of our model is that it does not yield immediate welfare conclusions, nor does it have obvious implications for the firm's market conduct. Our model is a first step towards a better understanding of some important aspects of the firm's financial decisions and we can only hope at this stage that future extensions and generalisations will allow us to gain new insights about the efficiency of the bankruptcy process.

As we pointed out earlier, one can give other interpretations to the contingent allocation-of-control rule described in section II. We have mainly emphasized the interpretation of bankruptcy so far, but one can think of other mechanisms that would correspond to this rule. We shall describe briefly another interesting application of our model to venture capital. It is sometimes argued that venture capitalists prefer to finance firms through a combination of loans, convertible preferred stock and common stock rather than financing the firm entirely through equity. One of the reasons, it is argued, is that there is the potential for conflict between the venture capitalist and the entrepreneur about the timing of the sale of the company. The venture capitalist is only concerned about the monetary return of the operation, whereas the entrepreneur may also be concerned about other factors such as reputation. The venture capitalist may wish to sell at an inopportune time for the entrepreneur. If this is of much concern to the entrepreneur, he will insist on obtaining a right of control over the decision to sell. One way of achieving this is to finance the firm partly through loans and debentures so as to give full control to the venture capitalist only when the performance of

the firm is bad and share control when the performance is good. (For an exposition of this point see Chimerine et al. [1987, pp.244-245]). In addition, a venture capitalist may sometimes wish to allow for the possibility of future financing by other investors. This additional funding may take the form of debt. Then it is possible that the venture capitalist may lose her claims if the firm is in a situation where it cannot repay its debt. To avoid turning all the rights over to these creditors in such an event, venture capitalists often invest in redeemable preferred stock (see Hoffman and Blakey [1987]).<sup>i</sup>



## REFERENCES

- Baird, D. [1986] "The Uneasy Case for Corporate Reorganisations," *Journal of Legal Studies*, Vol. XV, pp. 127-147.
- Baird, D. and T. Jackson [1985] Cases Problems and Materials on Bankruptcy.
- Bhagat, S. and J. Brickley [1984] "Cumulative Voting: the Value of Minority Shareholder Voting Rights," *Journal of Law and Economics*, Vol. XXII, pp. 339-365.
- Bray, M. [1986] Seminar notes, London School of Economics.
- Bulow, J. and J. Shoven [1978] "The Bankruptcy Decision," *Bell Journal of Economics*, pp. 437-454.
- Chimerine, L. R. Cushman and H. Ross [1987] The Handbook of Raising Capital: Financing Alternatives for Emerging and Growing Business, Dow-Jones, Irwin.
- Green, J. and J. Shoven [1982] "The Effects of Financing Opportunities and Bankruptcy on Entrepreneurial Risk Bearing," in Entrepreneurship, J. Ronen (ed.), Lexington, MA: Lexington Books.
- Gale, D. and M. Hellwig [1985] "Incentive Compatible Debt Contracts: the One-Period Problem," *Review of Economic Studies*, III, 647-664.
- Grossman, S. and O. Hart [1982] "Corporate Financial Structure and Managerial Incentives," in J. McCall (ed.), The Economics of Information and Uncertainty, Chicago: University of Chicago Press.
- Grossman, S. and O. Hart [1986] "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691-719.
- Hart, O. and B. Holmstrom [1987] "The Theory of Contracts" forthcoming in Advances in Economic Theory, Fifth World Congress, T. Bewley (ed.), Cambridge: Cambridge University Press.
- Hart, O. and J. Moore [1985] "Incomplete Contracts and Renegotiation," Working paper, London School of Economics.
- Hellwig, M. [1981] "Bankruptcy, Limited Liability, and the Modigliani-Miller Theorem," *American Economic Review*, Vol. 71, pp. 155-170.
- Hoffman, H. and J. Blakey [1987] "You Can Negotiate with Venture Capitalists," *Harvard Business Review*, pp. 16-24.
- Jackson, T.H. [1986] The Logic and Limits of Bankruptcy Law, Cambridge: Harvard University Press.



- Jensen, M. and W. Meckling [1976] "Theory of the Firm: Managerial Behaviour, Agency Costs and Capital Structure," *Journal of Financial Economics*, 3, pp. 11-25.
- Kitagawa, Z. (ed.) [1987] Doing Business in Japan, edited by Matthew Bender, NY.
- Miller, M.H. [1977] "Debt and Taxes," *Journal of Finance*, pp. 261-275.
- Modigliani, F. and M. Miller [1958] "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review*, 53, 261-297.
- Myers, S. [1984] "Presidential Address: the Capital Structure Puzzle," *Journal of Finance*, 575-593.
- Ross, S. [1977] "The Determination of Financial Structure: the Incentive-Signalling Approach," *Bell Journal of Economics*, 3, pp. 458-482.
- Ruster, B. [1987] Business Transactions in Germany, Matthew Bender (ed.), NY.
- Simeon, Moquet, Borde et Associates [1987] Doing Business in France, Matthew Bender (ed.), NY.
- Stiglitz, J. [1969] "A Reexamination of the Modigliani-Miller Theorem," *American Economic Review*, 59, 784-93.
- White, M. [1980] "Public Policy Toward Bankruptcy: Me-First and Other Priority Rules," *Bell Journal of Economics*, 550.
- White, M. [1984] "Bankruptcy, Liquidation, and Reorganisation," in Handbook of Modern Finance, (chapter 35), E. Logue (ed.).

Appendix 1: Sharing Rules Contingent on Actions.

Let  $s^i$  ( $i = 1, 2, 3$ ) denote the entrepreneur's share of second period returns when action  $a_i$  is taken in period 1. In our model we imposed the restriction:  $s^1 = s^2 = s^3$ . Now, if contracts can be made contingent on actions, it is easy to see that the first-best outcome can be attained through signing the following equity contract:

$$s^2 = 1; s^1 = s^3 \in (0, 1).$$

Suppose the investor controls the firm; then, since  $s^2 = 1$ , she will never choose action  $a_2$ . Furthermore, since  $s^1 = s^3$  and  $\pi_{g1} > \pi_{g3}$ , she will choose action  $a_1$  in the good state  $\theta_g$  and  $a_3$  in state  $\theta_b$  since  $\pi_{b3} > \pi_{b1}$ .

Therefore, the first-best pair of actions  $(a_1, a_3)$  can be implemented by transferring control to the investor. In particular, there is no role for bankruptcy in this case. This conclusion is not surprising with only two states of nature and three possible actions: by making the contract fully contingent on actions we circumvent the incompleteness with respect to the state of nature.

However, this conclusion is no longer true in a model with more than two states. For example, let us consider the following extension of our model, with three possible states of nature  $\theta_g^1, \theta_g^2, \theta_b$  and with the same actions  $a_1, a_2, a_3$ . In the bad state  $\theta_b$ , which occurs with probability  $(1-q)$ , the ranking of actions by both agents is the same as before and liquidation is the optimal action to be taken in that state. In state  $\theta_g^1$ , which occurs with probability  $q/2$ , the potential conflict between the entrepreneur and the investor is described by the following inequalities:

$$\pi_{\xi 2}^1 > \pi_{\xi 1}^1; \pi_{\xi 3}^1.$$

$$\pi_{g1}^1 - \ell_1^1 > \pi_{g2}^1 - \ell_2^1; \pi_{g3}^1 - \ell_3^1.$$

Finally, in state  $\theta_g^2$ , which can also occur with probability  $q/2$ , the preferences of both parties are given by:

$$\pi_{g1}^2 > \pi_{g2}^2; \pi_{g3}^2.$$

$$\pi_{g2}^2 - \ell_2^2 > \pi_{g1}^2 - \ell_1^2; \pi_{g3}^2 - \ell_3^2.$$

In this example, the first-best' may no longer be implementable by a contract contingent on actions. For instance, consider the contract defined above where  $s^2 = 1$  and  $s^1 = s^3$ . This contract makes sure that action  $a_2$  is never chosen by the investor. This was optimal in the previous set-up, but is no longer optimal here since  $a_2$  is the first-best action in state  $\theta_g^2$ .

Formally, the first-best is not implementable by giving all the control to the investor when the following constraints cannot all be satisfied:

$$(IR): \quad q/2(1-s^1)\pi_{g1}^1 + q/2(1-s^2)\pi_{g2}^2 + (1-q)(1-s^1)\pi_{b3} \geq k.$$

$$(IC)_{g1}: \quad (1-s^1)\pi_{g1}^1 \geq (1-s^2)\pi_{g2}^1 \\ \geq (1-s^3)\pi_{g3}^1$$

$$(IC)_{g2}: \quad (1-s^2)\pi_{g2}^2 \geq (1-s^1)\pi_{g1}^2 \\ \geq (1-s^3)\pi_{g3}^2$$

$$(IC)_b: \quad (1-s^3)\pi_{b3} \geq (1-s^1)\pi_{b1} \\ \geq (1-s^2)\pi_{b2}.$$

Similarly, for the same reasons as in the text, the first-best cannot be implemented by giving control to the entrepreneur when the following

constraints cannot all be satisfied:

$$(IR): \quad q/2(1-s^1)\pi_{g1}^1 + q/2(1-s^2)\pi_{g2}^2 + (1-q)(1-s^1)\pi_{b3} \geq k.$$

$$(IC)_{g1}: \quad s^1 \cdot \pi_{g1}^1 - \ell_1^1 \geq s^2 \cdot \pi_{g2}^1 - \ell_2^1 \\ \geq s^3 \cdot \pi_{g3}^1 - \ell_3^1$$

$$(IC)_{g2}: \quad s^2 \cdot \pi_{g2}^2 - \ell_2^2 \geq s^1 \cdot \pi_{g1}^2 - \ell_1^2 \\ \geq s^3 \cdot \pi_{g3}^2 - \ell_3^2$$

$$(IC)_b: \quad s^3 \cdot \pi_{b3} - \ell_3 \geq s^2 \cdot \pi_{b2} - \ell_2 \\ \geq s^1 \cdot \pi_{b1} - \ell_1.$$

Then we can extend the analysis developed in Section II and in particular restate Propositions 1 through 3.<sup>8</sup>

Appendix 2: Sharing Rules  $s_2(y_1)$  Contingent on First-period Returns.

The main purpose of this appendix is to extend Proposition 3 to the general case where the entrepreneur's share of second-period returns can be made contingent upon first-period revenue  $y_1$ :  $s_2 = s_2(y_1)$ .

First, consider a non-voting share contract (NVS) with sharing rule  $s_2(y_1)$ . For such a contract to implement the first-best pair of actions  $(a_1, a_3)$  [ $a_1$  in the good state  $\theta_g$ ,  $a_3$  in the bad state  $\theta_b$ ], it is necessary that for all  $y_1 \in [y, \bar{y}]$ , the following incentive constraints be satisfied:

$$(IC) \quad s_2(y_1) \cdot \pi_{g1} - \ell_1 \geq s_2(y_1) \cdot \pi_{g3} - \ell_3$$

$$s_2(y_1) \cdot \pi_{b3} - \ell_3 \geq s_2(y_1) \cdot \pi_{b1} - \ell_1.$$

(If  $s_2(y_1) = \hat{s}_2$  for all  $y_1$ , then  $\hat{s}_2$  must be at least greater than or equal to  $\frac{\ell_3 - \ell_1}{\pi_{b3} - \pi_{b1}}$  in order to implement  $(a_1, a_3)$ .)

Now, if the cost of liquidating the firm is large, i.e., if  $\hat{s}_2$  is large the investor will not sign such a NVS contract since:

$$(7) \quad (1 - \hat{s}_2) \cdot (q \cdot \pi_{g1} + (1 - q) \pi_{b3}) < k.$$

In this case the first-best cannot be implemented through signing a NVS contract. However, a NVS contract will implement the second-best pair of actions  $(a_1, a_1)$ . In Section II we saw that the optimal payoff that the entrepreneur can get through signing a NVS contract with constant sharing rule  $s_2(y_1) = \hat{s}_2$  was:

$$\pi_{NVS} = q \cdot \pi_{g1} + (1 - q) \pi_{b1} - k.$$

Now, by making  $s_2$  contingent on  $y_1$ , the entrepreneur can improve upon this payoff. The idea is to make use of the correlation between  $y_1$  and  $\theta$ :

Let  $(s_2, \bar{s}_2, \hat{y})$ , where  $s_2$  and  $\bar{s}_2 \in [0, 1]$  and  $\hat{y} \in (y, \bar{y})$ , be chosen in such a way that:

(a) the incentive constraints for the entrepreneur to choose action  $a_1$  in the good state and action  $a_3$  in the bad state are satisfied when  $s_2(y_1) \geq \bar{s}_2$ . (In particular  $\bar{s}_2$  must be at least equal to  $\hat{s}_2$ ).

(b) The following equality holds:

$$(8) \quad q \cdot [(1-\underline{s}_2) \cdot (1-F_g(\hat{y}))\pi_{g1} + (1-\bar{s}_2)F_g(\hat{y})\pi_{g1}] \\ + (1-q)[(1-\underline{s}_2)(1-F_b(\hat{y}))\pi_{b1} + (1-\bar{s}_2)F_b(\hat{y})\pi_{b3}] = k.$$

(We use the notations:  $F(y|\theta_g) = F_g(y)$ ;  $F(y|\theta_b) = F_b(y)$ ).

Now take the NVS contract defined by the following sharing rule:

$$s_2(y_1) = \begin{cases} \bar{s}_2 & \text{for } y_1 \in [y, \hat{y}] \\ \underline{s}_2 & \text{otherwise. } (\underline{s}_2 \text{ can be arbitrarily small}). \end{cases}$$

Given equality (8), such an NVS contract satisfies the investor's IR-constraint. Furthermore, the entrepreneur's payoff if such a contract is signed will be:

$$q \cdot \pi_{g1} + (1-q)[(1-F_b(\hat{y}))\pi_{b1} + F_b(\hat{y}) \cdot \pi_{b3}] - k > \pi_{NVS},$$

since  $\hat{y} > y$  and  $\pi_{b3} > \pi_{b1}$  by assumption.

Now, if we assume MLRP (i.e., that the ratio  $\frac{f(y_1, \theta_g)}{f(y_1, \theta_b)}$  is non-decreasing in  $y_1$ ), it is easy to prove that there exists an optimal NVS contract of the above  $(\underline{s}_2, \bar{s}_2, \hat{y})$  form.

Given the positive correlation between first-period return  $y_1$  and the state of nature, it is optimal to choose  $s_2(y_1)$  so as to satisfy the entrepreneur's IC constraints for the lower values of  $y_1$ , i.e., for  $y_1 \in [y, \hat{y}^{NVS}]$ , where  $\hat{y}^{NVS} \leq \bar{y}$ .

(The proof proceeds along the same line as the proof of Proposition 3: we can use MLRP to prove that an NVS contract that satisfies the



incentive-constraints -- i.e., implements the first-best pair of actions  $(a_1, a_3)$  -- for all  $y_1 \in I$  where  $I$  is not of the form  $[y, \hat{y}]$  cannot be optimal). In other words, it is optimal to choose  $s_2(y_1)$  so that:

- for  $y_1 \leq \hat{y}$ , the first-best pair of actions  $(a_1, a_3)$  is implemented.
- for  $y_1 > \hat{y}$ , the second-best pair  $(a_1, a_1)$  is implemented.

On the other hand, since both the entrepreneur and the investor are risk-neutral, there is no loss of generality in assuming that  $s_2(y_1)$  is constant over each subinterval  $[y, \hat{y}]$  and  $(\hat{y}, \bar{y}]$ : i.e.,

$$\begin{aligned} s_2(y_1) &= \underline{s}_2 \text{ for } y_1 > \hat{y} \\ &= \bar{s}_2 \text{ for } y_1 \leq \hat{y}. \end{aligned}$$

Let  $(\underline{s}_2, \bar{s}_2, \hat{y}^{NVS})$  be an optimal non-voting share contract.

(we have:  $\bar{s}_2 \geq \underline{s}_2$  and  $\hat{y}^{NVS} < \bar{y}$  when condition (7) holds).

We will now prove that such a contract is dominated by a debt contract where control is shifted away from the entrepreneur whenever  $y_1$  is low ( $y_1 \leq \hat{y}^D$ ). The argument is simple: consider the optimal NVS contract  $(\underline{s}_2, \bar{s}_2, \hat{y}^{NVS})$  and let us look at the following debt contract:

$$s_2(y_1) = \underline{s}_2 \text{ for all } y_1 \text{ and bankruptcy takes place for } y_1 \leq \hat{y}^{NVS}.$$

Such a contract implements the pair of actions  $(a_2, a_3)$  on the interval  $[y, \hat{y}^{NVS}]$  and the pair of actions  $(a_1, a_1)$  on  $[\hat{y}^{NVS}, \bar{y}]$ . Since  $\underline{s}_2 < \bar{s}_2$ , the investor's IR constraint becomes slack if such a contract is offered. In other words, such a debt contract is suboptimal, and in particular there exists another debt contract that implements the pair of actions  $(a_2, a_3)$  on a larger interval  $[y, \hat{y}^D]$  with  $\hat{y}^D > \hat{y}^{NVS}$ , and that satisfies the investor's IR-constraint. The optimal payoff that the entrepreneur can get through signing a debt contract is then:

$$\pi_D^* = q[F_g(\hat{y}^D) \cdot \pi_{g2} + (1-F_g(\hat{y}^D))\pi_{g1}] \\ + (1-q) \cdot [F_b(\hat{y}^D)\pi_{b3} + (1-F_b(\hat{y}^D))\pi_{b1}] - k.$$

On the other hand, the optimal payoff he could expect if he signs a NVS contract is:

$$\pi_{NVS}^* = q \cdot \pi_{g1} + (1-q)[F_b(\hat{y}^{NVS})\pi_{b3} + (1-F_b(\hat{y}^{NVS}))\pi_{b1}] - k.$$

We have:

$$\pi_D^* - \pi_{NVS}^* = q \cdot F_g(\hat{y}^D) \cdot (\pi_{g2} - \pi_{g1}) \\ + (1-q) \cdot (F_b(\hat{y}^D) - F_b(\hat{y}^{NVS})) \cdot (\pi_{b3} - \pi_{b1}).$$

$\hat{y}^{NVS}$  will necessarily be very close to  $\hat{y}$  whenever  $\hat{s}_2$  (i.e.,  $\bar{s}_2$ ) is close to one, e.g., when the cost of liquidation  $\ell_3$  is large. Then, provided a low return  $y_1$  is a sufficiently good signal of the bad state of nature occurring, the ratio  $\frac{F_b(\hat{y}^D) - F_b(\hat{y}^{NVS})}{F_g(\hat{y}^D)}$  will be large and  $\pi_D^* > \pi_{NVS}^*$ . This allows us to extend Propositions 2 and 3 to the general case where the sharing-rule  $s_2(y_1)$  is contingent on first-period returns  $y_1$ .

Q.E.D.

## Footnotes

<sup>1</sup>More generally, the entrepreneur's private costs may also depend on the state of nature. Then the private cost,  $\ell_{ik}$  denotes the entrepreneur's cost of taking action  $a_i$  in state  $\theta_k$ . Allowing for this extra degree of freedom increases the possibilities of obtaining a potential state-contingent conflict between the investor and the entrepreneur. The ideas developed in this paper do not depend on this more general representation of the entrepreneur's private costs. Consequently, we decided to stick with the simpler formulation of the entrepreneur's cost, where the cost of taking action  $a_i$  is the same in every state.

<sup>2</sup>A standard debt contract would also impose the following restrictions on second-period sharing rules:

$$s_2(y_1, y_2) \cdot y_2 = y_2 - R_2 \quad \text{if } y_1 \geq R_1 \text{ and } y_2 \geq R_2$$

and

$$s_2(y_1, y_2) \cdot y_2 = 0 \quad \text{if } y_2 \leq R_2 \text{ and } y_1 \geq R_1.$$

A standard equity contract would also impose:

$$s_2(y_1, y_2) = s_2 \quad \text{for all } y_1 \text{ and } y_2.$$

<sup>3</sup>A fourth possibility is, of course, not to allocate control to either of the parties. That is, to let the entrepreneur and investor bargain about what action to choose in period 1 (after the realisation of  $\theta$ ). A thorough discussion of this case is taken up in section III, where we look at contracts with renegotiation.

<sup>4</sup>Bankruptcy is by no means the only mechanism for transferring control from the entrepreneur to the investor. Whenever the entrepreneur must seek additional external funding for the firm he is giving up some control to these outside investors. The latter can usually make refinancing conditional on the entrepreneur choosing certain actions. In short, whenever the firm's cash flow constraints are binding, it may have to comply with the wishes of outside investors. This point is discussed at greater length in section IV.

<sup>5</sup>At least one constraint is always binding.

<sup>6</sup>Whether  $(a_1; a_1)$  or  $(a_1; a_2)$  are optimal action plans for the entrepreneur is unimportant for our purposes. What is important is that either action plan is different from the first-best plan and from the plan chosen by the investor, if she controlled the firm.

<sup>7</sup>If there were no limited liability, i.e., if the entrepreneur's share  $\hat{s}$  could take on negative values, then renegotiation would always lead to the first-best outcome when the initial contract gives full control to the investor.

<sup>8</sup>In Proposition 1, condition (b) becomes:

$$(b') \quad \min \{q/2[(\pi_{g1}^1 - \ell_1) - (\pi_{g2}^1 - \ell_2)], q/2[(\pi_{g2}^2 - \ell_2) - (\pi_{g1}^2 - \ell_1)]\} \\ > (1-q) \cdot [(\pi_{b3} - \ell_3) - (\pi_{b1} - \ell_1)].$$









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